FIBER-BUNDLE-CELLS MODEL OF TENSILE TESTING FABRIC SAMPLES

László Mihály VAS[1], Fatma GÖKTEPE[2]*, Péter TAMÁS[1], Marianna HALÁSZ[1], Lívia KOKAS PALICSKA[3]

[1] Budapest University of Technology and Economics, Faculty of Mechanical Engineering, 1111 Budapest, Műegyetem rkp. 3., Hungary
[2] Namik Kemal University, Çorlu Engineering Faculty, Textile Engineering Department, Turkey
[3] Óbuda University, Rejtő Sándor Faculty of Light Indrustry and Environmental Protection Engineering, 1034 Budapest, Doberdó u. 6., Hungary
E-mail: goktepef@gmail.com; vas@pt.bme.hu

ABSTRACT

The goal of this paper is to develop a structural-mechanical model for describing the tensile behavior of textile fabrics in different directions based on the fiber flow theory and the fiber-bundle-cells modeling theory and method. This expected to make it possible to analyze the tensile and shear deformation behavior needed for modeling the symmetric and asymmetric drape test behavior of fabrics with more accuracy.

Key words: tensile behavior of textile, modeling of textile, fiber flow theory, fiber-bundle-cells modeling, drape test behavior of fabrics

1 INTRODUCTION

The fibrous structures such as textile materials, the fiber reinforced composites, and the linear polymers, too, are built up of discrete fiber-like elements, textile or reinforcing fibers, or yarns as e.g. the element of fabrics. The adjoining fibers or those intersecting a cross section of a fibrous sample create certain small assemblies that are fiber bundles in which the fibers show collective group-behavior [1-4]. The fiber bundle can be treated as intermediate elements of a fibrous structure which can represent the statistical properties of the geometry or the strength.

Besides the classic one [1], L.M. Vas and al. [4, 5] have introduced some other idealized statistical fiber bundles called fiber-bundle-cells (FBC) and shown that they can be applied to modeling some structural and strength properties of fibrous materials.

In this paper is a FBC model for describing the tensile behavior of textile fabrics reinforced composites in main directions is presented and its applicability is demonstrated in the case of fabric reinforced membrane.

2 FIBER BUNDLE CELLS BASED MODELING METHOD

Statistical fiber bundle cells

Fibers in a fibrous structure can be classified according to their geometry (shape, position) and mechanical behavior (strain state, gripping). These fiber classes are called fiber bundle cells (FBCs) (Fig. 1) [4].

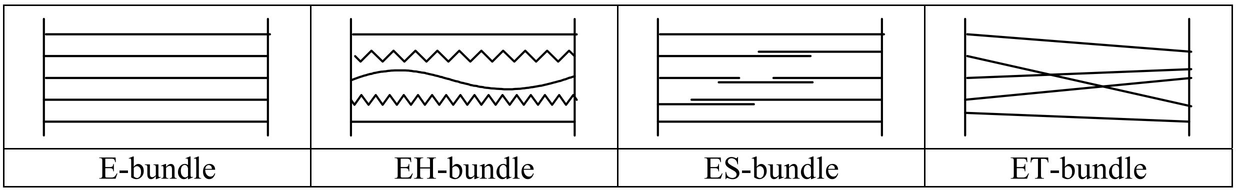


Fig. 1 Structural scheme of the idealized fiber bundle cells

Fibers of these FBCs are supposed to be perfectly flexible, linearly elastic and to break at a random strain (ε_S). They are straight in the E-bundle, loose (ε_o<0) or pre-tensioned (ε_o>0) in the EH-bundle, and oblique (fiber angle $\beta \neq 0$) in the ET-bundle, and gripped ideally in these cases. Fibers in the ES-bundle are straight but they may slip out of their grip at a strain level ($\varepsilon_b < \varepsilon_S$) or create fiber-chains with slipping bonds. Both the shape, position, and strength parameters of fibers are assumed to be independent stochastic variables.
Considering a constant rate elongation tensile test the tensile force ($F(u)$) creates a stochastic process as a function of the bundle strain (u). Being aware of the relationship between the bundle (u) and fiber strains (ε), the expected value of the tensile force of the FBCs ($E(F)=\overline{F}$) could be calculated as a sum of the single fiber forces using the suitable formulas developed [4]. Dividing the expected value by the mean breaking force of fibers, the normalized tensile force of bundle is computed as follows:

$$0 < FH(z) = \overline{F}(z)/n\overline{F}_S \leq 1, \qquad z = u/\overline{\varepsilon}_S \tag{1}$$

where n, $\overline{F}_S$, and $\overline{\varepsilon}_S$ are the number, the mean breaking force and strain of fibers, respectively, and z is the bundle strain normalized by the mean breaking strain of fibers. Fig.2 shows the graphic relationship between the strain of individual flexible fibers and the bundle. In case of ES-bundle ε_{bL} is the relative slippage way of fibers. In Fig. 3 the typical normalized expected value processes calculated at different parameter values are plotted for the FBCs. For the numerical calculations all random parameters were assumed to be of normal distribution.

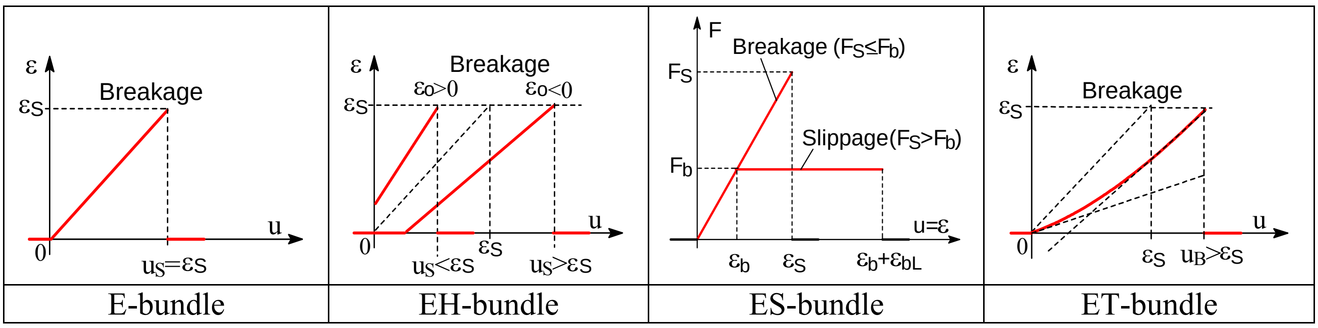


Fig. 2 Relationship between the strains of single fibers and FBCs

From Fig. 3 it is obvious that the FBCs can model rather complicated mechanical behaviors such as the initial convex part caused by crimped fibers (EH-bundle) or the slippages generated plateau beyond the peak (ES-bundle) even if they are used in themselves.

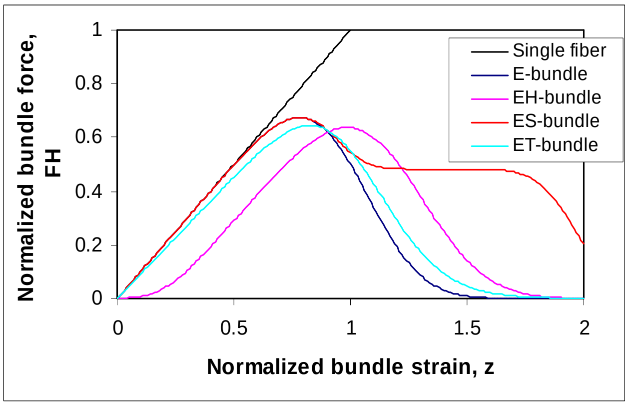


Fig. 3 Expected value of typical normalized force-strain curves of the FBCs

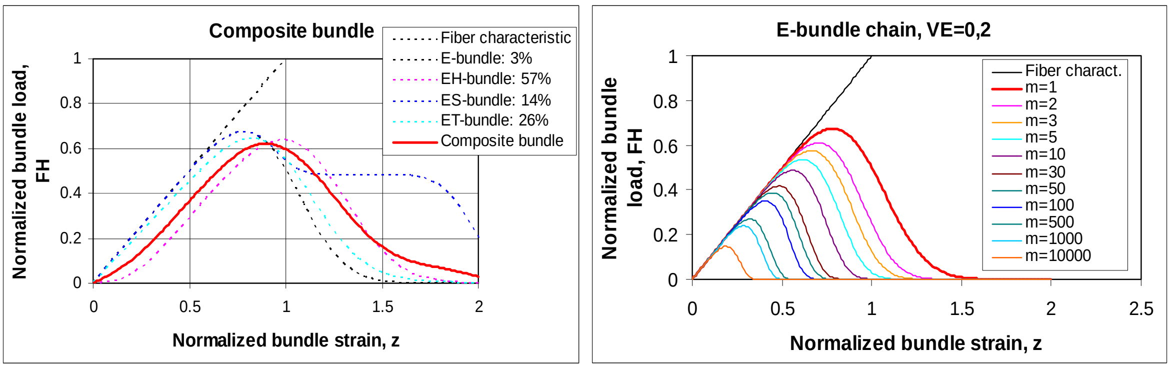
Parallel and serial connection of fiber-bundle-cells as FBC models

In general, several types of FBCs are needed to model the response of a real fibrous structure. In most cases the parallel connected FBCs (Fig. 4.a) called composite bundle provide a suitable model and the resultant expected value process is calculated as the weighted sum of the single FBC responses where the weights are the fiber number ratios [4,5]. In case the size effect such as the gauge length on strength are examined serial connection of the same type of independent FBCs is suitable to use creating a bundle chain (Fig. 4.b) [4, 5].

(a) (b)

Fig. 4 (a) Parallel and (b) serial connections of FBCs

As examples Fig. 5.a shows the weighted sum of the normalized force-strain curves visible in Fig. 3 (percentages are the relative weight values of FBCs) while in Fig. 5.b the effect of the number (m) of serial connected E-bundles is demonstrated causing the decrease of the peak value of the resultant force-strain curves that characterizes the tensile strength of the E-bundle chain (VE is the relative standard deviation of ε_S).



(a) (b)

Fig. 5 Normalized mean force-strain curves of parallel (a) and serial (b) connected FBCs

3 STRUCTURAL PROPERTIES OF FABRIC SAMPLES

Let us consider a rectangle sample with length L_o and breadth B_o cut out of the fabric in direction α where α=0° and α=90° are for the weft and warp directions, respectively (Fig. 6). Consequently, α is the angle of weft yarns to the length direction of the sample and the warp yarns are perpendicular to the weft direction (α+β=π/2).

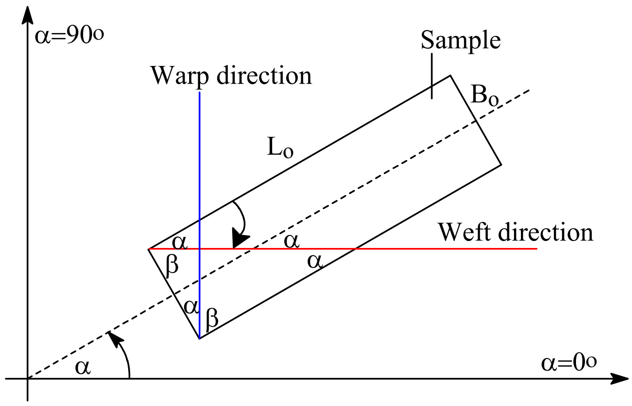


Fig. 6 Disposition of a sample to the main structural directions of the fabric

The fabric sample is for tensile test where it is gripped at the ends of B_o breadth and L_o (assuming $B_o \leq L_o$, as it is usual in practice) is the free or gauge length of that that is L_o does not include the parts needed for gripping as it can be seen in Fig. 7.

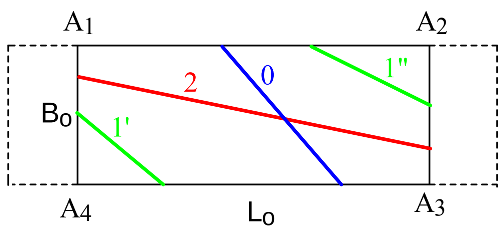


Fig. 7 Classification of yarns according to their gripping position

The edges of the gripped part of the specimen bounding the free length, $\overline{A_1A_4}$ and $\overline{A_2A_3}$, can be called gripping lines. The yarns in the sample show different mechanical behavior according to the number of their gripped ends at the gripping lines (Fig. 7):

- 2-gripped yarns are gripped at both of their ends;
- 1-gripped yarns are gripped at one of their ends;
- 0-gripped yarns are not gripped at any of their ends.

4 CONCEPT OF MODELING FABRIC SAMPLES USING FIBER-BUNDLE-CELLS

As a first step of modeling the tensile behavior of samples cut out in the main structural directions of the fabric will be tested and analyzed by using FBC modeling method neglecting the crimping of yarns as it is in the usual finite element layer models.

The expected value of the tensile force process of the E-bundle (Figs.1-3) related to a single yarn (in the direction of the tensile load) can be calculated by the following formula [4]:

$$\overline{F}(u) = E[F(u)] = \overline{K}u\left(1 - Q_{\varepsilon_S}(u)\right) \qquad (2)$$

where u is the bundle strain, $\overline{K}$ is the mean tensile stiffness of the yarns, $Q_{\varepsilon S}$ is the distribution functions of the breaking strain. The yarn parameters can be determined by tensile tests of yarn samples of gauge length L_o. Using the formulas according to Equation (1) for normalizing Equation (2) we obtain:

$$FH(z;L_o) = z\left(1 - Q_{\varepsilon_S}\left(z\bar{\varepsilon}_S(L_o)\right)\right) = \kappa(z)RH(z;L_o) \tag{3}$$

where the expected tensile characteristic (κ) and the reliability function (RH) of the E-bundle are defined by Equation (3).

All that is valid for gauge length L_o at which the tensile test is performed and it is well known that the tensile strength parameters of yarns depend on the gauge length [2, 3]. Supposing the gauge length is changed for $L=nL_o$ (n=1,2,…) and the section of fibers of length L_o create a so called bundle chain of independent elements (Fig.4.a) then the normalized expected tensile process of an E-bundle created by fibers of length L can be calculated as follows [6] (Fig. 5.b):

$$FH(z;L) = z\left(1 - Q_{\varepsilon_S(L)}\left(z\bar{\varepsilon}_S(L)\right)\right) = z\left(1 - Q_{\varepsilon_S(L_o)}\left(z\bar{\varepsilon}_S(L_o)\right)\right)^{L/L_o} \tag{4}$$

It can be noted, that formula (4) can be extended for $L<L_o$ as well. Consequently the following relationship can be obtained which is valid for both $0<L<L_o$ and $L\geq L_o$:

$$\left(\frac{\mathrm{FH}(z;L_o/n)}{z}\right)^n = \frac{\mathrm{FH}(z;L_o)}{z} \tag{5}$$

According to both modeling and experiences the mean breaking force of yarns and its standard deviation increase with reducing the gauge length which is known as size effect in the literature [3, 6].

5 APPLICATION OF THE FBC MODEL

5.1 Experimental

To demonstrate the applicability of the FBC model of fabric samples a plain woven cotton fabric and their yarn components were tested (Table 1).

No	**Material**	**Type**		**Density [g/m^2]**	**Type of weave**	**Yarn count**		**Yarn density λ [1/10mm]**		**Twist direction**	
		warp	**weft**			**warp**	**weft**	**warp**	**weft**	**warp**	**weft**
P	cotton	OE-rotor	OE-rotor	156	plain	Nm 34	Nm 34	26	22	Z	Z

Table 1 Data of the three examined fabrics

The weft and warp yarns were examined by tensile testing using the gauge length of 50 mm as it is used usually e.g. on the KES System. The test speed and the pretension were 12 mm/min and 0.2 cN, respectively. The results are summarized in Table 2.

Statistical properties	**Yarn** (Nm 34, Z twist)		
	Force peak [cN]	**Elongation peak** [mm]	**Strain peak** [%]
Mean	272.40	3.240	6.478
S.D.	32.31	0.349	0.697
C. of V. [%]	11.86	10.76	10.76

Table 2 Tensile test result of yarns

Finally tensile tests were carried out on fabric P samples cut out in main directions (weft and warp) with breadth of 50 using tensile tester Zwick Z50. The gauge length and the test speed were 50 mm and 12 mm/min, respectively.
Strength data are summarized in Table 3 where the mean indicates the data of average curve of the single measurements calculated point by point.

Fabric P - Breadth: 50 mm				
	weft_1	weft_2	weft_3	Mean
MaxF [N]	418.5	432.8	412.9	**380.9**
Δ*l* (maxF) [mm]	7.31	8.05	7.82	7.31
ε(maxF) %]	14.62	16.10	15.64	14.62
	warp_1	warp_2	warp_3	Mean
MaxF [N]	492.0	480.0	500.7	**472.4**
Δ*l* (maxF) [mm]	8.36	8.83	8.82	8.41
ε(maxF) [%]	16.72	17.66	17.63	16.82

Table 3 Tensile test results of fabric samples

5.2. Results of FBC modeling

The results of tensile measurements performed on fabric P in the main structural directions and its yarn component form the experimental background of the FBC modeling.

Cutting out a sample e.g. in weft direction from the fabric means the sample is built of weft yarns aligned lengthwise and warp yarns aligned crosswise (Fig. 8.b). Loading this sample in lengthwise direction the load is taken up by the 2-gripped weft yarns and the 0-gripped warp yarns play just a modifying role by interlacing the weft yarns even if densely. In the present paper the effect of interlacing is taken into account just by the adhesion between the yarns playing an important role in the cases of 1- or 0-gripped yarns but it can be neglected for the 2-gripped yarns.

Consequently, in a rough view, the sample cut out in main direction creates a bundle of 2-gripped yarns with an orientation angle of zero therefore it can be modeled by a simple E-bundle (Fig. 8.a).

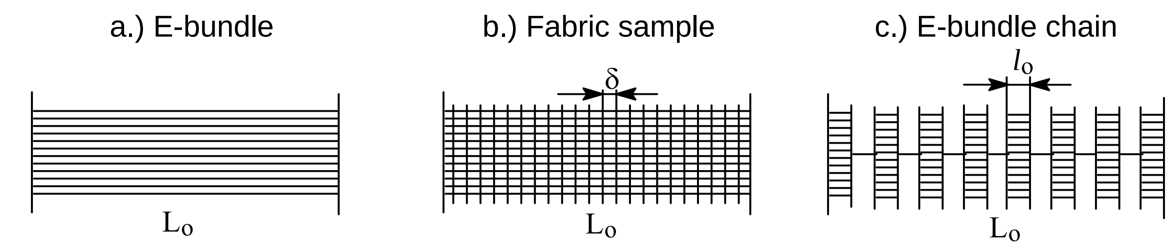


Fig. 8 E-bundle (a), a fabric sample cut out in main direction (b) and E-bundle chain (c) as the model of the sample

The normalized expected tensile force process related to a single yarn of an E-bundle is given by Equation (3). The number of yarns loaded is determined by the yarn densities (λ_{ok}, k=1,2) (Table 2) by which multiplying formula (3) provides the FBC estimation of tensile force recorded during a tensile test of a sample cut out in the weft (k=1) or warp (k=2) direction:

$$\begin{aligned} E[F(u)] &= E[F_k(u)] = \lambda_{ok} B_o E[F^{(k)} 1(u)] = \\ &= \lambda_{ok} B_o \bar{F}_S^{(k)} z\left(1 - Q_{\varepsilon_S}^{(k)}\left(\bar{\varepsilon}_S^{(k)} z\right)\right) = \lambda_{ok} B_o \bar{F}_S^{(k)} FH^{(k)}(z) \end{aligned} \qquad (6)$$

This is valid in the case of E-bundle built up of yarns of given length L_o.

FiberSpace is suitable to provide $FH^{(k)}(z)$ because this simple model does not contain any combined FBC. For modeling – in this simple case – just the mean and CV of the breaking strain of the yarn are needed which can be found in Table 2. They are respectively denoted by AE (=0.0648) and VE (=0.108) in Fiber Space.

The results of modeling E-bundle of L_o=50 mm yarns – which was imported in Microsoft Excel – can be seen in Fig. 10 ($L=L_o$=50 mm). This expected tensile force process can be approximated by averaging point by point measured and normalized force-strain curves. According to Fig. 10 the expected yarn strength efficiency (which is determined by the peak value of the normalized curve) in the fabric sample in main direction is 0.782 that 78.2%. This is valid for both weft and warp directions because of the identical weft and warp yarns and the symmetric structure of a plain weave.

The estimated expected tensile strength in both directions can be calculated by using Equation (6). As it is usual according to the related Standards the breadth of the sample was taken B_o=50 mm (Table 4).

	Estimation	**Weft direction**	**Warp direction**	**Fabric mean**
Yarn strength utilization [%]	real	78.2	78.2	78.2
Fabric tensile strength calculated for 50 mm width [N]	ideal	300	354	327
	ideal	234	277	256
Fabric tensile strength measured at 50 mm width [N]	----	381	472	427

Table 4 Results of FBC modeling using yarns of 50 mm length and measurements (mean)

The strength values in Table 4 are considered ideal when they were calculated as the simple product of the number of yarn in the direction of load and the mean yarn strength and the real ones were obtained by multiplying the latter by the expected yarn strength utilization provided by the modeling for bundle of yarns of 50 mm length (Fig. 8.a).

On the basis of Table 4 it can be stated that the measured tensile strength values proved to be larger then the ideal ones. This can be explained by two facts:

(1) The mean tensile strength of yarns strongly depends on the gauge length used (size effect); the smaller it is the larger the mean strength is [1-3].

(2) The crosswise yarns create a kind of gripping for the tensile loaded yarns sectioning them into short E-bundles which form a so called E-bundle chain (Fig. 8.c) [6]. The effective length of these bundles (l_o; Fig. 8.c) can be larger than the distance between the crosswise yarns (δ; Fig. 8.b) because of the possibility of some slippage and the strain of yarns at the peak force.

The mean strength of these short E-bundles can be much greater than that of the longer but the standard deviation of these short yarn segments is larger as well. The strength of this bundle chain is determined by the "weakest link" [2] yet this minimum value can be significantly larger than that of the original bundle of long yarns.

Regarding the breaking force only, suppose all the yarn breakages take place in a single short bundle (Fig. 9.a) meaning that the other bundles are subjected to strain only and the model of this behavior can be represented by a short E-bundle and a serial connected elastic continuum part (Fig. 9.b).

In this case the force-elongation relation is governed by the E-bundle and the role of the elastic part is to model the surplus in elongation as the contribution of the other bundles. In this model reaching the peak force value of the bundle the breakage of the chain can occur in a catastrophic way if the breaking bundle can not cover the loss in elongation after the bundle force peak [6]. These drops in force can take place after each other if the yarn breakages are distributed over several bundles of the chain.

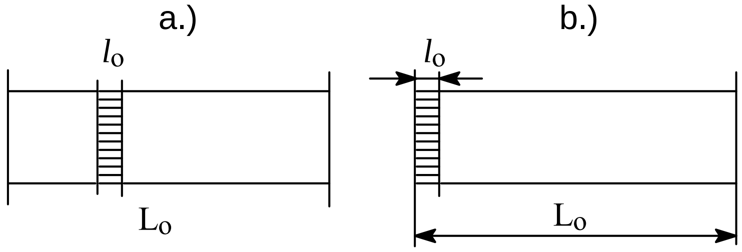


Fig. 9 Fabric sample as the serial connection of a single breaking E-bundle and an elastic part

As the other extreme case, the breakages of single yarns can be evenly distributed over the chain realizing an expected tensile process identical with that of the single bundles [6].
In the reality the damage and failure process is realized as one between the two extreme damage cases. In addition, the yarn chain that is a bundle chain which consists of a single yarn can be treated as a lower estimation of the real one [6].

In the sense of the “weakest link” concept Equation (4) describes the expected tensile force process of an E-bundle chain where the number of E-bundle is $n=L/L_o$. In this case the peak value and the half-width of the force-chain curve (corresponding to the standard deviation of the yarns) decrease by increasing n or L (Fig. 10). At the same time concerning the tensile strength the E-bundle chain can be considered as a single E-bundle built up of yarns of length L. In this sense Equation (4) can be used for bundles of yarns of length smaller than L_o as well. In the latter case the peak value and the half-width increase (Fig. 10).

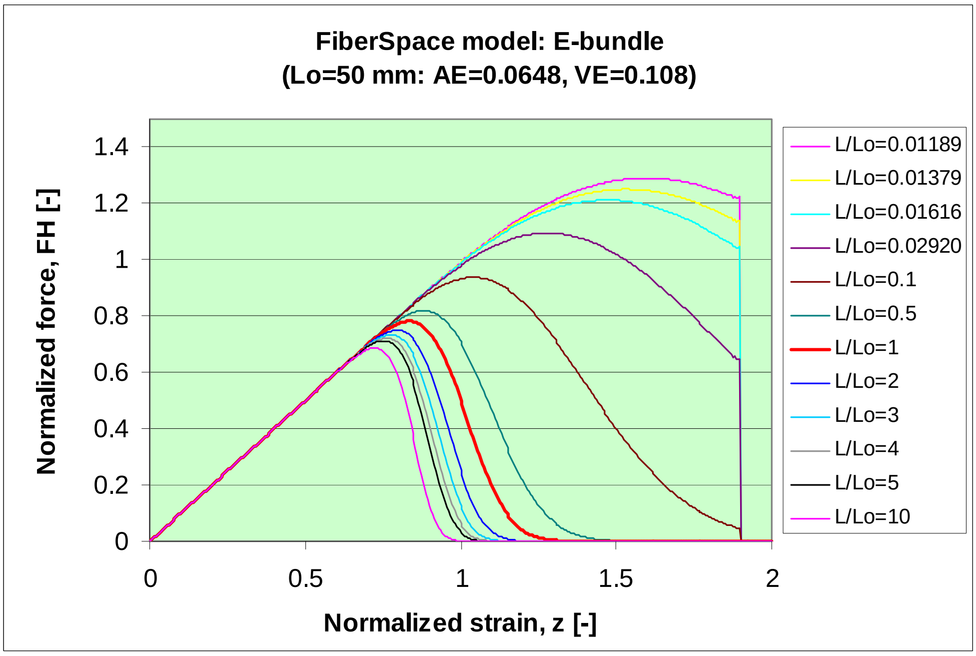


Fig. 10 Normalized expected force-strain curves of E-bundles with different relative lengths (L/L_o)

In this extended sense Fig. 11 shows the normalized peak force values as a function of the yarn length that is the gauge length of the yarns (L) the former in linear scale the latter in logarithmic scale.

The results of modeling discussed above give a good basis for analyzing the measurements obtained by tensile test of fabric samples. Since our simple FBC modeling method applies E-bundles only consequently the so called structural elongation caused by the

crimping of the yarns and the elastic pulling out of the grips are not modeled therefore the first step of the analysis is the determination of the structural elongation.
This defined by the steepest tangent straight line belonging to the inflexion point of the rising part of the force-elongation curve. The structural elongation is determined by the intersection point of the tangent and the elongation axis (Fig. 11).

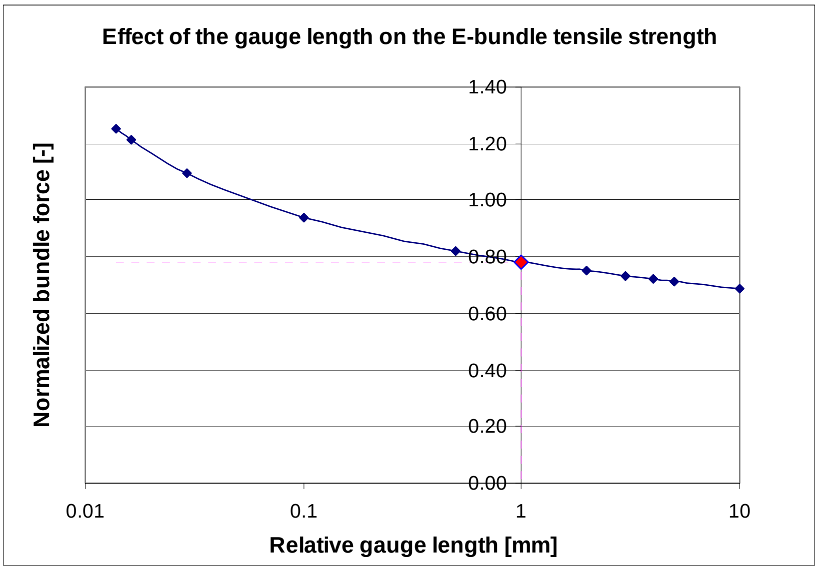


Fig. 11 Peak values of E-bundles versus relative yarn length in logarithmic scale

Figs. 12 and 13 show the measured and averaged force-elongation curves (blue lines) and the steepest tangents.

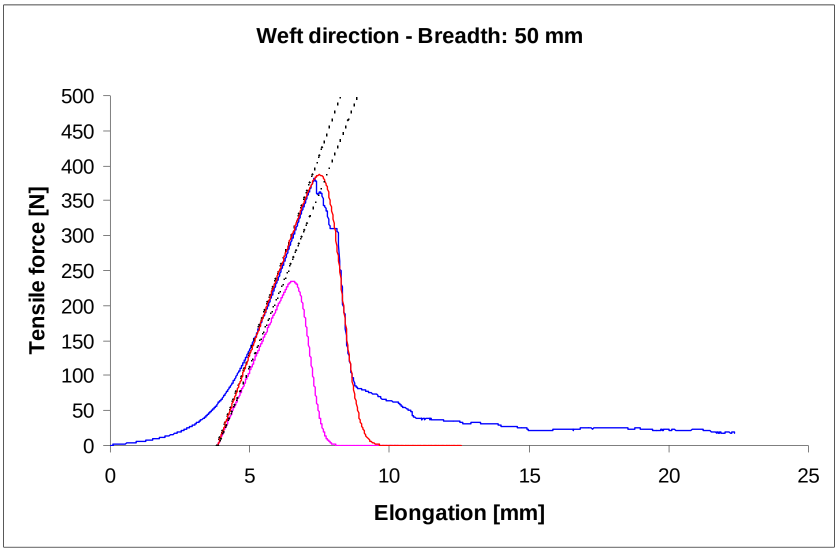


Fig. 12 Measured and averaged force-elongation curve (blue line), shifted E-bundle curve (lilac line), and the transformed model curve (red line) for samples cut out of weft direction

In these diagrams the E-bundle model curves (lilac lines) with their initial tangent are shifted from the origin by the structural elongation.
It can be seen in the diagrams (Figs. 12, 13) that the linear variable transformation (that is applying the proper scaling) of these shifted E-bundle curves (red lines) fit well to the measured ones regarding both the rising and the falling branches of the curves.
The results of measurements, modeling, and model based analysis are summarized in Table 5.

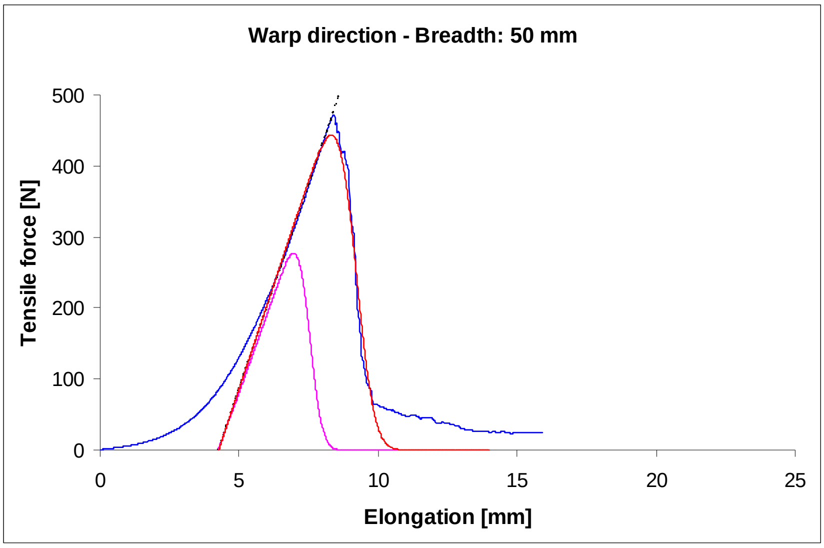


Fig. 13 Measured and averaged force-elongation curve (blue line), shifted E-bundle curve (lilac line), and the transformed model curve (red line) for samples cut out of warp direction

Sample width L_o [mm]	Origin of result	Properties	Weft	Warp
50	Measured	Size of weave cell δ [mm]	0.4545	0.3846
		Tensile strength [N]	380.9	472.4
		Tensile stiffness [N/mm]	113	115
		Structural elongation [mm]	3.85	4.26
	Modeled by yarns	Max force [N]	234.4	277.0
		Tensile stiffness [N/mm]	100	115
		Yarn strength utilization [%]	78.21	78.21
	Shifted and scaled model	Scale factor of elongation [-]	1.35	1.50
		Scale factor of force [-]	1.65	1.60
		Max force [N]	386.7	443.2
		Yarn strength utilization [%]	1.29	1.2514
		Length ratio, $n=l_o/L_o$	0.011898	0.01379
		Effective bundle length, l_o [mm]	0.5945	0.6897
		Relative eff. bundle length, l_o/δ [-]	1.31	1.79

Table 5 Results of FBC analysis based on E-bundle model

8 CONCLUSIONS

On the basis of the diagrams and numerical results some essential statements can be done.

(1) The structural elongation caused by the interlacing and crimping of the yarns adding to that the elastic pulling out of the grips is rather large (about 8%).

(2) The large structural elongation means that it is important to take into account of describing the drape behavior of the fabrics.

(3) The yarn strength utilization of ideal E-bundle consisting of independent yarns of 50 mm length is 78.2% which is relatively small. The measured utilization was larger than 100% meaning that the interlaced crosswise yarns bind together the segments of the loaded yarns forcing them strong working together and by that a relatively small effective length is realized much smaller than the gauge length of the fabric sample.

(4) The effective bundle length values (l_o) determined by the shifted and rescaled E-bundle curves using Equation (4) are larger than the weave cell sizes (δ) indicating

that the crosswise interlacing yarns can slip on the loaded ones causing the increase of the bundle length.

(5) The slippage of interlacing yarns which is in relation with certain friction and shear effects indicates that it can be also important factor for modeling the drape behavior of fabrics.

E-bundle based modeling of the tensile strip test of fabrics in the main directions can be well used for analyzing the tensile measurements and the structural-mechanical behavior of fabric specimens tested.

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